

**THREE DIMENSIONAL FINITE ELEMENT METHOD  
APPLIED TO DIELECTRIC RESONATOR DEVICES**

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**ABSTRACT**

Three dimensional finite element method (F.E.M.) is applied to evaluate electromagnetic and electrical parameters of the  $TE_{01\delta}$  cylindrical dielectric resonator (D.R.) mode housed into a parallelipipedic metallic enclosure.

Numerical results concerning both frequencies, field vectors and coupling coefficients between adjacent D.R. are presented.

**I - INTRODUCTION**

A number of methods have been proposed for the determination of electromagnetic parameters of cylindrical D.R. [1-8]. In most of these methods, the enclosures in which the D.R. are inserted are supposed cylindrical and the excitation is not taken into account.

But in practice, the D.R. will be housed in a rectangular metallic enclosure and it becomes necessary to carry out the analysis taking into account the effects of: the enclosure - the supporting substrate - the presence of the input and output coupling lines.

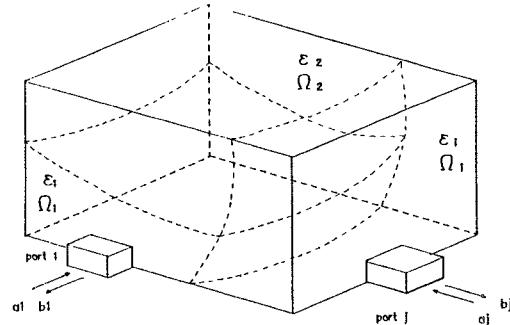
To overcome these difficulties, we propose to solve the forced oscillation wave equation by means of the three dimensional F.E.M.

A particular case of this problem is the free oscillation one when the excitation is out of consideration. In this last case, eigen modes can be determined.

This paper contains the first results obtained for free oscillation cylindrical D.R. working on its fundamental  $TE_{01\delta}$  mode housed into a parallelipipedic microstrip box. The results presented concern both: resonant frequencies, vector fields and coupling between two adjacent D.R.

**II - GENERAL FORMULATION**

The geometry of the inhomogeneous metallic structure that we have to study is given in figure 1.



- Figure 1 -

An example of metallic enclosure composed of dielectric medium  $\Omega$

The dielectric material contained into it, is assumed to be linear, lossless, isotropic, and of arbitrary shape.

Dividing the volume into a number of second harmonic tetrahedrons and using the vectorial F.E.M., a three components magnetic field  $H$  formulation is developed to solve the following equation [3-4-5] :

$$\iiint_V \left( \frac{1}{\epsilon_i} (\vec{\text{rot}} H) \cdot (\vec{\text{rot}} H^*) \right) dV - K_0^2 \iiint_V (\mu_i H) \vec{H}^* dV = \\ = \sum_{j=1}^n \iint_{S_j} (\vec{n}_j \wedge \left( \frac{1}{\epsilon_i} \vec{\text{rot}} H \right)) \cdot \vec{H}^* dS_j \quad (1)$$

in which

$H$  is the magnetic field vector

$\epsilon_i$  is the relative dielectric constant,  $i=1,2,\dots$

$\mu_i$  is the relative permeability,  $i=1,2,\dots$

$K_0 = \omega_0 \sqrt{\epsilon_0 \mu_0}$

$\vec{n}_j$  is the normal vector of section  $S_j$  at port  $j$

Note that the second member of this equation cancels when we consider free oscillations.

Discretisation and resolution of system (1) yield resonant frequencies and the components of magnetic field of the given mode.

As three components vector formulation is used, the spectrum of eigensolutions contains spurious responses. The following means are suggested to reduce and identify them :

- imposing effective boundary conditions respectively on metallic (or magnetic) walls and on geometric symmetries of the system, the number of undesirable modes decrease.

- physical solutions are characterized by a weak magnetic divergence in front of numerical spurious modes

- introducing the term  $[D \iiint_V (\vec{\nabla} \cdot \vec{H}) (\vec{\nabla} \cdot \vec{H})^* dV]$  in first member of equation 1 and then solving it we can distinguish physical and spurious modes for some real parameters D [6].

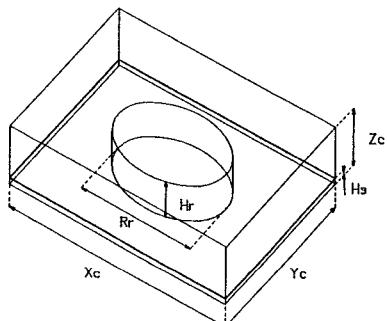
- field lines graph drawn from the E.M. field components permits also to characterize physical modes

### III - RESULTS APPLIED TO D.R.

Computation is conveyed in portable standard programs written in MODULEF normes [7]. It has been implemented on the following examples :

- D.R. excited on  $TE_{01\delta}$  mode

We consider a cylindrical resonator of radius  $R_r$  and height  $H_r$  placed on a supporting dielectric substrate of thickness  $H_s$  inside a perfectly conducting parallelepipedic cavity of basis dimensions  $X_c \times Y_c$  and length  $H_c$  (figure 2). The D.R. and substrate permittivities are respectively  $\epsilon_r$  and  $\epsilon_s$ .



- Figure 2 -

Shielded dielectric resonator including substrate  
 $R_r=6 \text{ mm}$     $H_r=6 \text{ mm}$     $\epsilon_r=36$   
 $H_s=1 \text{ mm}$     $\epsilon_s=2.2$   
 $X_c=24 \text{ mm}$     $Y_c=24 \text{ mm}$     $H_c=10 \text{ mm}$

Symmetries of the  $TE_{01\delta}$  mode are exploited to reduce the amount of computation time required. So only a quarter of the general structure is considered where enclosure planes are supposed to be electric walls. The computation of the resonant frequency ( $F_0$ ) of  $TE_{01\delta}$  mode

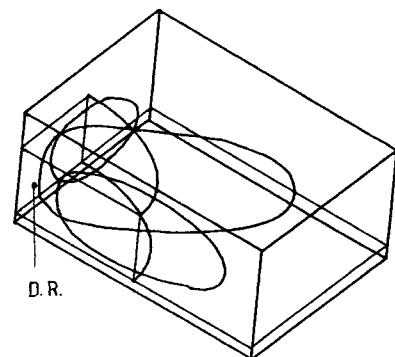
of D.R. enclosed in this structure is :

$$F_0 = 4.8475 \text{ GHz}$$

Note that the two dimensional F.E.M. gives a resonant frequency  $F_0=4.8486 \text{ GHz}$  for a D.R. housed in a cylindrical enclosure of radius 12 mm.

Accuracy and computation time required depend on number of elements the volume is parted (1860 tetrahedrons - 3045 nodes to obtain the precedent result).

Magnetic field lines of this mode  $TE_{01\delta}$  have also been drawn and are given in figure 3.



- Figure 3 -  
Magnetic field lines of a shielded dielectric resonator

- Effects of a microstrip line

The three dimensional F.E.M. permits also to take into account the presence of the microstrip line near the D.R. on its resonant frequency and on its magnetic field repartition.

The studied structure and its dimensions are presented on figure 4.

For a same system and an identical partition of it volume (1152 tetrahedrons-1993 nodes),  $TE_{01\delta}$  resonant frequencies modes are :

- without transmission line

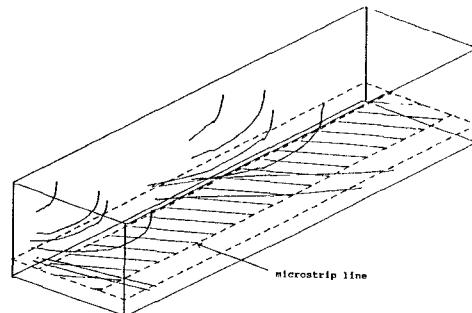
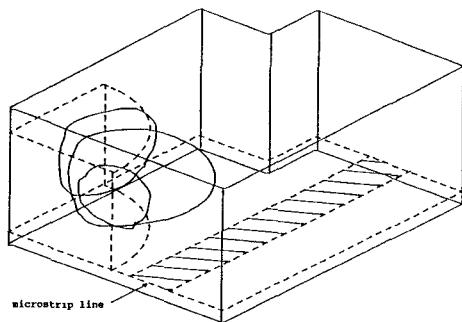
$$F_0=5,119 \text{ GHz}$$

- with transmission line  $F_0=5,135 \text{ GHz}$   
The effect of the presence of the microstrip line on magnetic field lines of the D.R. is presented on figure 5.

- Coupling coefficient between adjacent D.R.

The calculation of the coupling coefficient between two adjacent D.R. excited on their  $TE_{01\delta}$  modes has also been driven. It is a fundamental parameter for several applications, in particular in microwave filters one's.

Now in the methods generally given, we never consider simultaneously, the presence

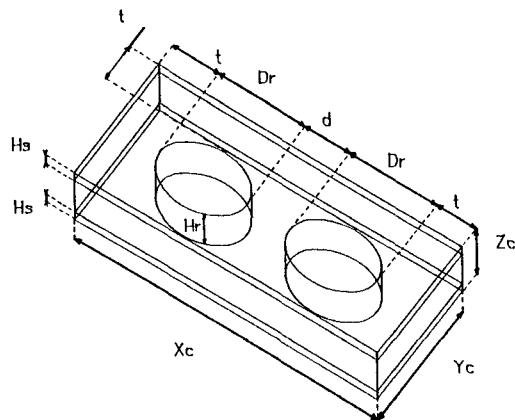


(a)

- Figure 4 -

Magnetic field lines of a shielded dielectric resonator including substrate and upper armature of a microstrip line

$$\begin{aligned}
 Rr &= 6 \text{ mm} & Hr &= 6 \text{ mm} & \epsilon_r &= 36 \\
 Hs &= 0,79 \text{ mm} & \epsilon_s &= 2,2 \\
 Xc &= 24 \text{ mm} & Yc &= 24 \text{ mm} & Hc &= 10 \text{ mm}
 \end{aligned}$$



- Figure 6 -

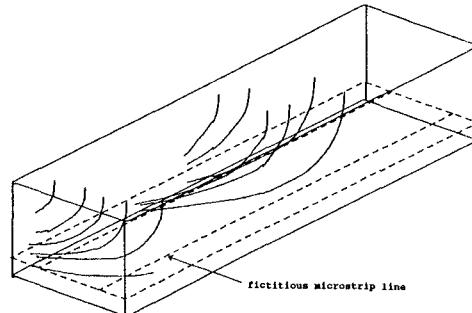
Coupling between two shielded D.R.

$$\begin{aligned}
 Rr &= 6 \text{ mm} & Hr &= 6 \text{ mm} & \epsilon_r &= 36 \\
 Hs &= 1,5 \text{ mm} & \epsilon_s &= 2,2 \\
 Xc &= 42 \text{ mm} & Yc &= 24 \text{ mm} & Hc &= 9 \text{ mm} \\
 d &= 6 \text{ mm} & t &= 6 \text{ mm}
 \end{aligned}$$

of the two D.R. The three dimensional F.E.M. permits to analyze rigorously the problem.

Two adjacent identical D.R. are enclosed in a perfectly conducting parallelepipedic cavity between two dielectric substrates. The distance separating the two D.R. is  $d$ . The walls of the structure are at a distance  $t$  from the D.R. ends (figure 6).

Using symmetries and solving the boundary value problem for the field and resonant frequency in this structure, we can perform the coupling



(b)

- Figure 5 -

Magnetic field line around microstrip line  
a - with metallic upper armature -  $Fo=5,131$  GHz  
b - without upper armature -  $Fo=5,119$  GHz

calculation  $Ko$  between two D.R. excited on their  $TE_{01\delta}$  mode [4]  $Ko$  satisfies [2] :

$$Ko = \frac{Foe^2 - Fom^2}{Foe^2 + Fom^2}$$

where  $Foe$  is the resonant frequency of the even mode

$Fom$  is the resonant frequency of the odd mode

For a structure like that given in figure 6, we obtain :

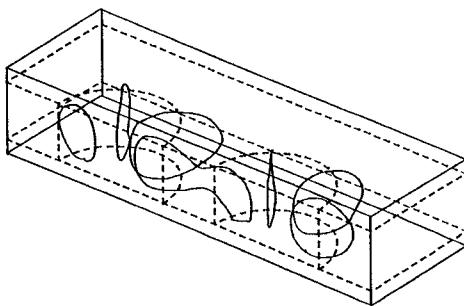
$$Fom = 5.157 \text{ GHz}$$

$$Foe = 5.184 \text{ GHz}$$

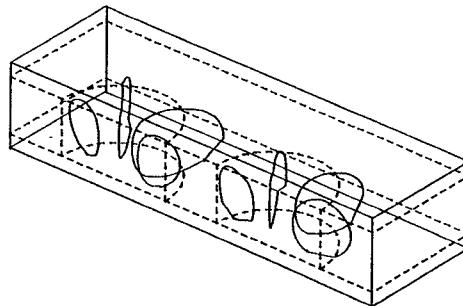
$$Ko = 5.22E-03 \text{ (1752 tetrahedrons - 2856 nodes)}$$

Computational coupling coefficient  $Ko$  agrees with experimental value ( $Kexp$ ) since  $Kexp=4.80E-03$ .

Magnetic field lines for a distance of a 6 mm between the two D.R. corresponding to "even and odd modes" are presented in figure 7.



(a)



(b)

- Figure 7 -  
**Coupling between two D.R.**  
 a - low frequency -  $F_{om} = 5.157$  GHz  
 b - high frequency -  $F_{oe} = 5.184$  GHz

#### IV - CONCLUSION

The used of 3D F.E.M. has permit to obtain magnetic fields, resonant frequencies and coupling coefficient of D.R. acting on their dipolar mode and inserted into a structure which can be used for experimentation. No approximations have been introduced and the method presented here can be extented to study other modes like  $TM_0$  or HEM ones.

This method is now also extented to compute the scattering parameters of D.R. coupled with transmission lines.

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